TALKS

Caroline Accurso and David Siy (DeSales University)
On Colorizing Forest-Like Structures - Part I: Hailin' from Halin (Talk)
Given a table B with k rows of non-negative integers, we say that B is *colorizable* if there exists an edge-colored graph G where for each $i \in \{1, 2, 3, \dots k\}$, the degree sequence of the color i subgraph of G corresponds to row i of B. Although asking whether or not B is colorizable is in general an NP-Complete problem, the conditions required for determining whether or not B is colorizable as a forest are known. This motivates us to explore conditions for the colorizability of forest-like structures, such as Halin graphs and cactus graphs.

In Part I of this talk, we discuss Halin graphs in specific. A *Halin graph* is a tree T with a cycle connecting the leaves of T in the order determined by a planar embedding of T. A *partial Halin graph* is a spanning subgraph of a Halin graph. We present the degree sequences of Halin graphs as characterized by Türker Biyikoğlu and our extension of his results to partial Halin graphs. Finally, we demonstrate how these results advance us in our search for colorizability conditions of Halin graphs.

Bethany Baker, Laura Schlesinger, and Jean Springsteen (Coe College)
Spin-a-Block: A Game with Infinite Loops (Talk)
Spin-a-Block is an impartial, two-player game that allows for infinite loops. Each edge of the square blocks has one of two colors. Players alternate spinning a block 90° clockwise or counterclockwise. A game is won when all adjacent edges of the board have the same color. We strongly solved simple games, classifying them by Generalized Sprague-Grundy values and determined optimal strategies for gameplay.

Leah Bayer, Elise Eckman, and Zack While (Youngstown State University)
Evaluating Kostant's Multiplicity Formula: Part I (Talk)
Central to the study of the representation theory of Lie algebras is the computation of weight multiplicities, which are the dimensions of vector subspaces called weight spaces. The multiplicity of a weight can be computed using a wellknow formula of Kostant that consists of an alternating sum over a nite group and involves a partition function called Kostants partition function. There are two major obstacles in the use of this formula. First, the number of terms arising in the sum grows factorially as the rank of the Lie algebra increases and, second, the value of the partition function is often unknown. In this paper, we specialize to the Lie algebra slr+1(C). and focus on questions regarding the number of terms contributing nontrivially to Kostants weight multiplicity formula. Through this study, we show that these contributing sets, called Weyl alternation sets, show interesting combinatorial and geometric properties. We dedicate a section to detailed examples that illustrate accessible techniques students may use to begin investigating the open problems we present in this area.

Leah Bayer, Elise Eckman, and Zack While (Youngstown State University)
Evaluating Kostant's Multiplicity Formula: Part II (Talk)
Central to the study of the representation theory of Lie algebras is the computation of weight multiplicities, which are the dimensions of vector subspaces called weight spaces. The multiplicity of a weight can be computed using a wellknow formula of Kostant that consists of an alternating sum over a nite group and involves a partition function called Kostants partition function. There are two major obstacles in the use of this formula. First, the number of terms arising in the sum grows factorially as the rank of the Lie algebra increases and, second, the value of the partition function is often unknown. In this paper, we specialize to the Lie algebra slr+1(C). and focus on questions regarding the number of terms contributing nontrivially to Kostants weight multiplicity formula. Through this study, we show that these contributing sets, called Weyl alternation sets, show interesting combinatorial and geometric properties. We dedicate a section to detailed examples that illustrate accessible techniques students may use to begin investigating the open problems we present in this area.

Cristiana Bonds, Dustin Ford, and Craig Steele Jr. (Philander Smith College)
Examining Apparent Slumps in National Basketball Player Association Player Free Throw Shooting in the Playoffs (Part I)(Talk)
In sports statistics research, the hot hand phenomenon has received a great deal of attention with free throwing shooting as a typical event of interest, specifically streaks of made free throws. In recent years,

streaks of missed free throws have been receiving more attention in the media; however, very little has been done to determine if streaks of unsuccessful free throw attempts or slumps are truly unusual. This work is an examination of a subset of the 2015-2016 free throw data of the NBA. Here the Multiple Window Scan Statistic is used to investigate the statistical significance of slumps in free throw shooting by a select group of NBA players during the 2015-2016 season.

Vishnu Boyareddygari (Western Kentucky University)
Using Latin Hypercube Sampling and Partial Rank Correlation Coefficients to Assess Sensitivity in Mathematical Models (Part II) (Talk)
Mathematical models of wound healing inform researchers about the mechanisms involved and, in turn, the best path of treatment. Continued analysis of these models guides the refining process, leading to a more accurate model. Latin Hypercube sampling (LHS) is a stratified multidimensional sampling method used to produce a sampling of a parameter space that follows a more representative distribution than a sample produced through simple random sampling. Partial Rank Correlation Coefficients (PRCC) regression analysis of the outputs produced from substituting LHS values into the model under investigation allows for the correlation between a parameter and the model to be determined independently of the other parameters and assesses, along with the probabilistic p-value, the sensitivity of the parameters in the model. This method has been applied to a wound-healing neovascularization partial differential equation (PDE) model formulated by Schugart et al. (2007) in an attempt to determine which parameters have the strongest influence on the model output. This can aid in the determination of factors that are the most important in positively or negatively affecting the healing response.

Michael Boyle, Channing Parker, and Mary (Molly) Rowland (James Madison University)
KMC Dynamics: The Evolution of Crystal Structures (Talk)
Mathematical modeling has become very important and relatively cheap tool for understanding and developing new materials, which are necessary for technological applications.
We consider a solid-on-solid atomistic model for epitaxial growth. Atoms evaporate from, condensate on, and diffuse (hop) across the crystal surface. Facets reflect a long-range order of the atoms on the crystal surface. To explore the mathematics of facet formation, we numerically simulate a one-dimensional discrete silicon on silicon model. We then generalize this to a two-dimensional silicon on silicon model. We want to analyze the geometry of the emerging structure, and whether a facet evolves as a consequence of this atomistic model.

Myles Bradley (Coe College)
A Mathematical Model of Coral Reef Response to Destructive Fishing Practices with Predator-Prey Interactions (Talk)
Coral reefs are being degraded by multiple anthropogenic stressors, including excessive and destructive fishing practices. Such activities damage reefs directly, particularly when cyanide and explosives are employed, and deplete reef fishes that keep coral predators and competitors in check. In this work, we focus on the highly problematic corallivore Crown-of-thorns starfish (CoTS), *Acanthaster planci*, and one of its few known predators, the endangered and overfished Humphead wrasse, *Cheilinus undulatus*. We built a system of nonlinear ordinary differential equations to model the interactions between coral, wrasse, and CoTS biomasses within the Indonesian province of Raja Ampat. We consider commensalism between wrasse and coral in favor of wrasse, and predator-prey relationships between wrasse and CoTS, and CoTS and coral. We take into account coral damage from illegal, unregulated, and unreported (IUU) fishing and consider constant yield, constant effort, and seasonal wrasse harvesting. Equilibria for the system with and without harvesting are determined, including coexistence equilibria in which all three species persist. We run numerical simulations and conduct sensitivity analyses on key parameters. Through this work, we hope to provide insight on the extent to which the coral reefs of Raja Ampat can hold up to rising fishing pressure as well as describe a model which can be applied to similar ecosystems.

Kevin Chang (Williams College)
Kostant's Weight Multiplicity Formula and the Fibonacci and Lucas Numbers (Talk)
Kostant's weight multiplicity formula (KWMF) is an alternating sum indexed by a finite group, called the Weyl group, and involves a partition function. This formula yields the dimension of a weight space of a finite-dimensional Lie algebra in a highest weight complex irreducible representation. Our work focuses on

describing and enumerating the elements of the Weyl group which contribute non-trivially to KWMF for certain special highest weights. We prove that in some cases the number of terms contributing is given by the Fibonacci and Lucas numbers.

Emma Christensen, Nick Joslyn, and Maddy Kersten (Simpson College)
Optimization of Down Syndrome Specialty Care Clinic Locations using Operations Research (Talk)
As the field of healthcare continues to grow and evolve, specialized and coordinated care for extraordinary conditions becomes more and more necessary. Down syndrome, one of these extraordinary conditions, is a chromosomal disorder that is often accompanied by other severe medical conditions that affect an individual's overall health. Currently, there are 71 Down syndrome specialty care clinics in 34 states in the U.S. that serve less than 10% of the eligible population. Using operations research, specifically p-median and maximal covering models, we found that the current placement of these 71 clinics adds to the problem of inaccessibility. Our research presents the optimal locations for Down syndrome specialty care clinics in the continental United States and offers potential future clinic locations given the current placement.

Matthew Costa and Kristen Godinez (California State University, Channel Islands)
A closer look at the Macomb County political party swing (Part I) (Talk)
In the 2012 presidential election, Macomb County in Michigan voted predominantly Democratic. However, a major switch took place in the 2016 election. The county primarily voted Republican, which was one of the key factors leading to a win by the Republican party. In order to analyze the election results we considered a system with two political party classes, Democratic and Republican, and took an epidemiological approach to model the movement of voters between the two parties. We also used a system of ODEs and performed stability analysis to delineate the results of the 2016 election. In this talk, we discuss the similarities between the spread of an infectious diseases and the spread of political ideologies. Furthermore, we compare our numerical results to data collected from Macomb County.

Eric Culver(Utah State University) *Losing positions with a minimum number of lines in the game of tP_3-saturator* (Talk)
The game of F-saturator involves two players taking turns drawing line segments between pairs of a finite number of points while trying to avoid creating the forbidden structure F. A losing position in this game is a collection of points and lines that does not contain F, but such that when any line is added, F is created. The forbidden structure we will study is t copies of a "path on three points", an object we call P_3; P_3 consists of three points, not necessarily collinear, connected by two line segments.

More formally, we will study the structure of tP_3-saturated graphs via the impartial combinatorial game. Although this talk is self-contained, it builds on the talk "*An Introduction to the Game of F-Saturator*" by Julie Sánchez. (Joint work with David Brown, Julie Sánchez, and Brent Thomas.)

Ty Danet, Garrett Lopez, and Angel Ramos (California State University, Channel Islands)
An Epidemiological Math Model Approach to a Political System with Three Parties (Part II) (Talk)
The United States has proven to be and remains a dual political party country. The 2000 and 2016 election has brought with them the growing number of voters moving into a third party. The goal of this research is to look at the relationships between political parties and their members. We analyze where people move between political parties, and the population dynamics amongst the voting class. To evaluate these attributes, we will approach the issue using a nonlinear mathematical model employing epidemiological methods with the assumption of a constant population that is homogeneously mixed. This approach should generate insight into how populations move from one political party to another, and at what point do rising third parties make a significant contribution to the political structure/voting system. We will determine the equilibria analytically and shall discuss the stability of the system, parameters will be expressed to stabilize a co-existence between three parties, and numerical simulations will be performed to verify and support analysis.

Leslie Decker, Park Mikels, and Andrea Van Wyk (Simpson College)
Modeling the Spread of Palmer Amaranth Across Iowa and the Midwest Cornbelt (Talk)
The Palmer Amaranth Think Tank, a group of undergraduate students, located at Simpson College in Indianola, Iowa, is focused on researching the spread of Palmer amaranth across the Midwest Corn Belt. In order to demonstrate how serious of a problem Palmer amaranth may be in the future, we used an agent-based simulation software called NetLogo. Starting in the counties confirmed to have Palmer amaranth, we

showed how the weed spreads across the Corn Belt through machinery, wind, and places where manure is used, such as hog confinements. Each land unit has a field type of corn, soybean, CRP, or none. We will also talk about Palmer amaranth seed discovered in CRP mixes. Lastly, we will discuss using the Briar Score to verify our model.

Benjamin Dulaney, Jenny Frank, and Dylana Wilhelm (James Madison University)
KMC simulations for homoepitaxial and heteroepitaxial crystalline structures (Talk)
The goal of our project is to model the surface growth of crystalline structures. We consider the cross-section of a crystalline film, which is deposited on a crystalline substrate. We use a solid-on-solid bond counting model where a surface atom undergoes one of three events: hopping, evaporation, or condensation. We utilize experimentally-verified rates, obtained by ab initio calculations, to predict the occurrence of an event at a given site along the film. We perform kinetic Monte Carlo simulations to describe the evolution of the crystal over time and to study how temperature influences the mobility of surface atoms. Finally, we extend the model to include heteroepitaxial growth, in which elasticity plays an important role.

Rachel French, Jacob Menix, and Rachel Turner (Western Kentucky University)
Using Parameter Estimation Techniques to Analyze a Mathematical Model in Wound Healing (Talk)
Since the medical treatment of diabetic foot ulcers remains a challenge for clinicians, a quantitative approach using de-identified patient data and mathematical modeling can help researchers understand the physiology of the wounds. The goal of this project is to utilize individual patient data to identify key parameters, through the use of nonlinear mixed effects modeling, and delayed rejection adaptive metropolis (DRAM), in the healing process in order to improve patient care and diagnosis. In this work, we plan to use nonlinear mixed effects modeling to attribute wound healing variability to either fixed effects, parameters that are more likely to remain constant for all patients, or random effects, parameters that vary from patient to patient. The identified random effects should then be taken into special consideration when treating patients with chronic wounds, especially diabetic foot ulcers. A Bayesian approach using DRAM, a MATLAB implementation of a Metropolis-Hastings algorithm, will also be used to estimate and analyze parameters. Using DRAM can help with parameter estimation since the available data is sparse and can help find parameter identifiability issue.

Rachel French and Carson Price (Western Kentucky University)
Using Latin Hypercube Sampling and Partial Rank Correlation Coefficients to Assess Sensitivity in Mathematical Models (Part I) (Talk)
Latin Hypercube sampling (LHS) is a stratified multidimensional sampling method. Partial Rank Correlation Coefficients (PRCC) regression analysis of the LHS values, along with the probabilistic p-value of the sampled parameter, assess the sensitivity of the parameters. This method has been used in conjunction with a wound-healing ordinary differential equation (ODE) model formulated by Krishna and Pennington (2015). Sensitivities of parameters are studied as a function of varying sampling technique and measured healing response of grouped patients.

Steven Giangreco and Allie Plunk (Missouri University of Science and Technology)
Modeling the Relationship Between Age, Genetics, and White Matter Microstructures (Talk)
The human brain's structure can change with age and different health conditions. A genetic risk factor called the epsilon 4 (e4) allele of the apolipoprotein E (ApoE) gene is linked with increased risk of Alzheimer's disease and reduced brain health. This study investigates whether healthy individuals with the e4 allele exhibit specific brain structure differences. Diffusion tensor imaging (DTI) was used to measure five different aspects of white matter structural integrity (e.g., fractional anisotropy) in seven different brain regions. A multivariate analysis of covariance (MANCOVA) was performed to test for differences in white matter structures and e4 status while controlling for age. Additional statistical methods were employed to test for demographic differences by e4 status and to control the family-wise error rate across multiple tests. These results will enable researchers to better understand the association between the e4 risk factor, age, and white matter structural differences in specific brain regions.

Alejandra Gutierrez and RaQuedra Lee (Philander Smith College)
An Examination of Above Average Performance Clusters in Professional Bowling (Part II)(Talk)

The hot hand phenomenon has been greatly discussed. Being hot is defined as having successes on the majority attempts at goal in a designated time frame. As a part of the discussion about the hot hand phenomenon, some research has emerged investigating the statistical significance of perceived high performance in bowling. This research has been limited to cases where the data had to be collapsed to two categories, one representing only a success characterized by a strike and one representing a failure characterized by a non-strike. This limitation results in an inaccurate portrayal of high performance in that it suggests that high performance is only characterized by a strike. We overcome this limitation and make available more complete results for the general sports enthusiast and the Professional Bowlers Association (PBA) league. This could potentially aid professional bowlers and the PBA league in a more accurate assessment of athletes performances. We assume that bowling data follows a multi-state, higher-order Markovian model, which allows us to collapse the data into three categories: high performance, average performance, and poor performance. We extract data from the PBA website and compute the multiple window scan statistic associated with perceived high performances. Then we use the exact distribution of the multiple window scan statistics to obtain the probability that under a particular set of conditions, an observed cluster is statistically significant. From this, our goal is to separate high performance clusters that may appear intuitively significant from those that could have happened by random chance.

Elizabeth Hollen and Jason Viehman (Missouri University of Science and Technology)
Statistical Analysis of DNA Methylation Data in a Cervical Cancer Study (Talk)
DNA methylation occurs when methyl groups attach to cytosine bases on DNA segments. Previous studies have established links between specific methylation patterns and many diseases. In this research, statistical methods are employed to test for significant differences in methylation levels between HIV patients with different stages of cervical cancer. DNA methylation levels are measured at cytosines across the genome with Illumina 450K methylation microarrays. After initial pre-processing to eliminate low-quality data, testing was performed at each cytosine site using t-tests and empirical Bayes tests to identify any statistically significant site level methylation differences between the cervical cancer stages. Two region level statistical methods (Bumphunter and DMRcate) were also applied to identify statistically significant regions of interest in the genome. Significant sites or regions that overlap with genes, CpG Islands, or other genomic annotations can help researchers better understand the molecular impact of DNA methylation and its connection to cervical cancer.

Michael Holmblad (Winona State University)
Understanding Elliptic Curves in Cryptography (Talk)
The need for technological security is needed for every device out there. My Computer and Network Security professor once told me, There are a couple hundred people in the world that truly understand Cryptography. To me, that number is too small. Elliptic Curve Cryptography is one of the most popular forms, but it is also very complex. It works through abstract algebra, computational power, and an elliptic curve function. Elliptic Curve Cryptography is one of two major forms of public key cryptography. Between it and RSA is, Elliptic Curve is much faster and more secure. It also has a trapdoor that as of right now is not vulnerable to sub-exponential attacks. Understanding Elliptic Curve Cryptography is no trivial matter. The goal is to present a ni. It will discuss Elliptic Curve Algebra, properties of the algebra, and applying it all in practice.

Arjun Kanthawar and Nikhil Krishna (Western Kentucky University)
On the Control and Identifiability of Models in Wound Healing (Talk)
Chronic wounds such as diabetic foot ulcers are the leading cause of non-traumatic amputation in developed countries. In order for researchers to better understand the physiology of these wounds, a mathematical model describing oxygen levels at the wound site can help predict healing responses. Daulton (2013) used optimal control theory to formulate a three variable differential-equation model to optimize hyperbaric oxygen treatment strategies. In the first part of this talk, we discuss a differential equation model with four variables adding a chemoattractant to Daultons model to better describe the healing response of the wound. In the second part of this talk, we analyze a different model using practical identifiability. In order to formulate a mathematical model that accurately represents the physiology of a wound, the model and its parameters must be identifiable when given actual data. Practical identifiability is a method used to determine whether parameters in a model can be uniquely determined given actual data. We are working with a differential equation model that describes the interactions among matrix metalloproteinases, their

inhibitors, the extracellular matrix, and fibroblasts (Krishna et al., 2015). Our approach uses a Fisher information matrix to find unidentifiable parameters using their coefficient of variations. We then find different subsets of these parameters that form identifiable combinations. We use a profile likelihood method to determine the algebraic relationship between different parameters.

Devin Lawson and Anna Marek (Coe College)
Minimal Clues for Unique End View Puzzle Solutions (Talk)
We studied End View puzzles, which are empty $n \times n$ boards with m clues of an alphabet of size p around the border. End View puzzles are solved when each row and column of the board is filled with a single instance of each element of the board's alphabet, and when looking in from a clue on the outside of the board, the first letter seen in that row or column must be that clue. We discovered and named different strategies for solving. Additionally, we examined the minimal clues required to find unique solutions for boards of different sizes and what boards have no unique solutions.

Francisco Mancia (University of North Texas at Dallas)
Virtual Path-width and bridge index of virtual links (Part I) (Talk)
We study the techniques from graph theory to develop new invariants of virtual knots and links. We define the planar graphs associated to a virtual link, and analyze their properties for the classification of virtual links. In particular, we calculate the virtual path-widths of a graph and analyze its relation to the virtual bridge number of a link. We prove that the virtual bridge index and virtual path-width of a pseudo prime knot are equal.

Nicholas Meyer (Winona State University)
On the Algebra of Rotations in $\mathbb{R}^3$ (Talk)
The need to represent rotations of objects in 3-D Euclidean space arises daily in many fields: animation, computer vision, and physics, to name a few. Ever since Euler first described his eponymous angles, without giving a tractable method for constructing them, mathematicians have longed for a better system to describe rotations. In 1843, William Rowan Hamilton had an epiphany whilst walking across Brougham Bridge in Dublin with his wife. Therein he inscribed the laws defining the quaternions, forever changing the face of rotations. The quaternions, when limited to having unit norm, form a group under multiplication which is isomorphic to SU(2). This presentation will discuss the interplay between these two groups and will clarify the use of quaternions to represent rotations. We will delve into the relationship between SU(2) and SO(3).

Cielo Perez and Aesha Siddiqui (Williams College)
Kostant's Partition Function for the Lie Algebra of Type A_3 (Talk)
Kostant's partition function counts the number of ways to write a given weight (vector) as a nonnegative integral sum of positive roots (a fixed set of vectors). We provide closed formulas for Kostant's partition function for the classical Lie algebra of type A_3 consisting of traceless 4×4 matrices with complex entries.

Ayush Prasad (Western Kentucky University)
Constructing an Optimal Design Method in a Mathematical Model for the Interactions of Matrix Metalloproteinases and Their Inhibitors in a Wound (Talk)
Because the medical treatment of diabetic foot ulcers remains a challenge for clinicians, a quantitative approach using patient data and mathematical modeling can help researchers understand the physiology of the wounds. In this work, we estimate parameter values using individual patient data curve-fitted to a modified version of a mathematical model that describes the interactions among matrix metalloproteinases, their inhibitors, extracellular matrix, and fibroblasts at a wound site (Krishna et al., 2015). The model and parameter values were then analyzed using global and local sensitivity analyses, which were used to describe how sensitive each parameter value of the model was to changes in the system. However, these model parameters can be estimated more efficiently and accurately by implementing an optimal design method that calculates optimal observation times for collecting clinical data. We introduce an SE-optimal design (standard error optimal design) by using a Fisher Information Matrix (FIM) to determine time evolution of sensitivity values. The goal of this work is to quantify and understand differences between patients to predict future responses and individualize treatment for each patient.

Julie Sánchez (Utah State University)
An Introduction to the Game of F-Saturator (Talk)

Euler and Gauss play a game on a finite number of points in the plane. They take turns drawing one line segment at a time between pairs of points, with at most one line segment between any pair. Each is trying to force the other to draw F, two line segments connected to any single point; he who draws F loses the game.

More formally, Euler and Gauss are playing an *impartial combinatorial game*, and therefore Euler may have a strategy that will always force Gauss to draw the forbidden structure, or vice-versa.

In this talk, we introduce a variety of impartial combinatorial games played on graphs with various Fs. In some cases, strategies are explained that guarantee a win to a certain player.

This talk, although not necessary, is a prequel to the talks "*Further Results on the Game of P_3-Saturator*" by Jessica Wilkinson and "*Losing Positions in the Game of tP_3-Saturator*" by Eric Culver. (Joint work with David Brown, Eric Culver, Sid Tate, Brent Thomas, and Jessica Wilkinson.)

Nicholas Speranza and Henry Wickus (DeSales University)
On Colorizing Forest-Like Structures - Part II: The Many Points of Cactus Graphs (Talk)
In Part II of this talk, we discuss the colorizability of another forest-like structure, namely, cactus graphs. *Cactus graphs* are connected graphs whose blocks are either cycles or edges, and *partial cactus graphs* are unions of cactus graphs. We give the characterizations of the potential and forcible degree sequences of partial cactus graphs. We then use these results to explore necessary conditions for the colorizability of cactus graphs.

Stefan Stryker (Western Kentucky University)
A Numerical Algorithm for a Partial-Differential-Equation Model of a Bacterial Infection in a Wound (Talk)
Oxygen therapy can provide aid for patients with chronic wounds. Investigating a mathematical model of the wound healing process can provide improvements for oxygen therapy. Our investigation is using a finite volume approach to numerically solve a system of partial differential equations that represent densities of bacteria, neutrophils, oxygen, and chemoattractant; a flux-limiting approach for the advection term is being used. Incorporation of this work into an optimal control model is the planned future direction of this research.

Sid Tate(Utah State University) *Entropy and Digraphs* (Talk)
Claude Shannon developed the concept now known as 'Shannon entropy' as a measure of uncertainty or disorder in information states. Given a discrete probability distribution $p = (p_1, ..., p_n)$, *Shannon entropy* is defined as $S(p) := \sum_{k=1}^{n} p_k \log_2 \frac{1}{p_k}$. The theory of entropy has been extensively applied to quantum mechanics: The entropy of a specific quantum state is defined as the Shannon entropy of its corresponding density matrix, in which the spectrum of its eigenvalues is viewed as that state's probability distribution. Recognizing a connection between these quantum states and undirected graphs as quantum state spaces, recent work in graph theory involves analyzing the entropy of undirected graphs. We further these ideas by applying the theory of entropy to directed graphs, in particular those that model pairwise comparisons also known as *tournament digraphs*. In order to circumvent the trouble of complex eigenvalues and whatever could be meant by complex probabilities, we employ a generalized notion of the Shannon entropy, the *Rényi α-entropy*, which is defined to be $H_\alpha(p) := \frac{1}{1-\alpha} \log_2 \left(\sum_{k=1}^{n} p_k^\alpha \right)$. Specifically, we determine the Rényi α-entropy of tournament digraphs using the eigenvalues of their adjacency matrices, and observe connections between their structure and maximal/minimal Rényi-α entropy. (Joint work with David Brown.)

Paul Wiggins (University of North Texas at Dallas)
Virtual Path-width and bridge index of virtual links (Part II) (Talk)
We study the techniques from graph theory to develop new invariants of virtual knots and links. We define the planar graphs associated to a virtual link, and analyze their properties for the classification of virtual links. In particular, we calculate the virtual path-widths of a graph and analyze its relation to the virtual bridge number of a link. We prove that the virtual bridge index and virtual path-width of a pseudo prime knot are equal.

Jessica Wilkinson(Utah State University) *Further Results on The Game of P_3-Saturator* (Talk)
When would you bet $1 million on a game? When is winning not only likely, but guaranteed? We consider a game where two players take turns drawing lines between points. The loser is the first player to create a point connected to two lines. This structure is called a path on three vertices, or a P_3. We will analyze

games in which one of the players has an unbeatable strategy. Although this talk is self-contained it builds off of "*Introduction to the Game of F-Saturator*" by Julie Sánchez, and delves further into the world of Graph-Saturation and impartial combinatorial games. (Joint work with Dave Brown, Eric Culver, Julie Sánchez, Sidney Tate, and Brent Thomas.)

Yimin Wu (Grinnell College)
Rank-Awareness in Compressed Sensing (Talk)
Last summer we examined common algorithms in the field of Compressed Sensing that are used to solve underdetermined systems of equations denoted by AX=Y, where X is a row-sparse matrix meaning most rows contain only zeros. Algorithms whose success rate of recovering X increases as the rank of X increases are defined as rank-aware algorithms. In this talk, we will exhibit how a specific Thresholding algorithm is rank aware, and we will chronicle our approach to modeling the behavior of Thresholding algorithms.

POSTERS

Gabriel Baedaro and Dalten Cross (Simpson College)
Finding Potential Automobile Buyers with Data Modelling (Poster)
When an independent person is attempting to sell an item, most of the responsibility of finding a buyer typically falls to the buyer. The seller makes a listing for the item, and it is then up to the buyer to determine what aspects of an item they want and to find the appropriate listing. However, sellers may want more control over the match-making process to get that item sold. Specifically, a seller may want to know which potential buyers are the most likely to purchase the item so that he can reach out to the buyer and make the sale. For this project, we attempt to predict which consumers are the most likely to buy a vehicle based on the automobile's own specifications with data modelling. With successful models, these proactive sellers can know which potential buyers would be the most responsive to these sales pitches.

Bethany Baker (Coe College)
Minimal Packings of Double Hexagon Tiles on Pseudo-Rectangular Boards (Poster)
A packing is a tiling of a board such that there are no overlapping tiles and another tile cannot be placed. We examine minimal packings of double hexagon tiles on pseudo-rectangular boards and establish the smallest number of double hexagon tiles needed for a minimal packing of all finite boards.

Andy Becker and Geoff Converse (Simpson College)
Predicting the Stock Market via Twitter (Poster)
Predicting the stock market has been a research topic which has stumped economists for years. Traditionally, economic indicators and company financials have been used to analyze the market, with varying results. With the introduction of Big Data, research opportunities for understanding the connection between public opinion and the stock market have opened. Our project goal is to use Machine Learning techniques to analyze Twitter data to predict the movement of a particular stock. When looking at specific companies, we are able to acquire upwards of 80,000 tweets per hour. We can compare the content of tweets between various days in order to understand the relationship between common and irregular Twitter data with stock movement. However, this process is high-dimensional, and it is computationally infeasible to determine the impact of individual tweets. Our solution is to aggregate our data into two dimensions by generating summary variables such as the emotion level (with natural language processing), outreach, and number of tweets to name a few. Once this data simplification process is completed, we will be able to easily determine the predictive power of Twitter using models such as K-nearest neighbors, neural networks, or support vector machines. We hope to find that general trends on Twitter are predictive for movement in the stock market.

Tyger Bottenfield (Simpson College)
Math and Billiards (Poster)
The purpose of this project is to be able to predict and demonstrate the relationships between a target (pocket) as well as the paths and movements involved with striking a pool ball. The actual demonstration will be shown in a simulation created using the program NetLogo. In order to reach this goal, it's essential to understand the geometric properties of varying shapes of billiards tables, as well as the properties of the lines created by the movements of a pool ball.

Hannah Carlson (Simpson College)
The Fach System: A Mathematical Model (Poster)
The Fach system is a method of categorizing operatic voices based on a number of factors such as range, tessitura, and vocal weight/color. The current system categorizes singers in a qualitative manner, usually with the help of a vocal teacher. However, it is my objective to create a more quantitative system by creating a mathematical model that can be used to determine vocal Fach using six aspects of the singers voice: their range, tessitura, passaggi, vibrato speed, frequency, and vocal color.

Christopher Colahan (Simpson College)
A history of Cryptography and Cryptanalysis (Poster)
We all use cryptography every day to secure our private information. Since the invention of the first ciphers millennia ago, ciphers have been continuously created and broken. The history of cryptography and cryptanalysis is examined, starting with the transposition and monoalphabetic substitution ciphers of the ancient Greeks and Romans. Next, frequency analysis is examined as a technique to break monoalphabetic and polyalphabetic substitution ciphers. Finally, the effect of modern computers, one-way hashes, and important ciphers are examined.

Allison Frideres (Simpson College)
Numerical Modeling, Simulation, and Parameter Estimation with Water Quality Data (Poster)
Tainter Lake is a reservoir located in Northern Dunn County, Wisconsin, that has experienced algal blooms due to a cycle of burgeoning cyanobacterial populations during the summer months. These blooms cause the water to become toxic and noisome thus posing problems for the local economy and quality of life surrounding the lake. A mathematical model was desired that could be used to test the feasibility and effectiveness of potential solutions to the algae problem. Previously collected REU and DNR data was examined in conjunction with local weather and flow data. We found that Tainter Lake has features similar to previously modeled lakes but it also differs significantly. In order to account for these differences, statistical models had to be created to specifically account for the conditions of Tainter Lake. The statistical models yield parameter estimates for a mathematical model created to forecast bloom conditions and severity.

Taylor Gehrls and Ashtyne Madsen (Simpson College)
Exploring the Game of Set (Poster)
The game of Set was invented by Marsha Jean Falco in 1974 when she was trying to understand if certain traits lead to epilepsy in German Shepherds. Today, Set is a popular game that tests the players visual perception and has rather simple concepts, but a complex mathematical structure. Set is made up of 81 unique cards that differ in shape, color, number, and shading. There are three options for each of these attributes, and sets are found among three cards when the attribute options are all different or at most have three the same. In our research, we have taken a closer look at calculating the probabilities of not drawing a set in a certain number of cards and the maximum number of cards that can be present without a set using modular arithmetic, Euclidean geometry, and Java programming. We then compare these results with an expanded version of the game, adding a fourth option to each attribute and drawing sets of four cards.

Orlando Guerra and Park Mikels (Simpson College)
Simplifying Knots Through Algorithmic Procedures (Poster)
In knot theory, knots can be represented by grid diagrams. The measure of a grid diagram‘s complexity is its size. In order to simplify a grid diagram, one must use a destabilization grid move, as it transforms the size of the knot‘s grid diagram from n to $n1$ while preserving the knot type. We wrote a program that takes in a grid diagram, represented by a two-dimensional array, and reduces it to a simplified grid diagram. The program automatically searches for stabilizations in the grid diagram and performs a destabilization grid move. If no stabilization is found, then the program determines whether commutations or switches will lead to a stabilization and then performs the destabilization.

Madeline Kersten (Simpson College)
Modeling School Climate and Anti-Bullying Programs in American Public Schools using Agent-Based Modeling (Poster)
Bullying is a pertinent issue in American public schools, which teachers and administrators across the nation are actively attempting to combat. Bullying negatively affects the overall school climate in a school system

because bullying impacts the four categories of school climate, namely safety, relationships, teaching and learning, and external environment. School administrations attempt to implement anti-bullying programs to varying degrees of success. Based on data from a middle school in Massachusetts, this study seeks to discover what effects anti-bullying programs have on school climate. Using agent-based modeling, a simulation of various anti-bullying programs can be achieved.

Sara Nielsen (Simpson College)
Data Analysis with Decision Trees (Poster)
The amount of data in the world today is increasing significantly and with that we have made huge advancements in technology to handle all this data. For this project I have been researching the affects that big data has made on our ever changing world. I have also been looking into data analysis tools, specifically decision trees. Decision trees are one of the most popular data analysis tools because they are easy to follow and interpret and consistently produce a predicatively accurate model. Many algorithms have been developed for decision trees including ID3, C4.5, and CART. I used each of these algorithms to create a decision tree for the same data set to compare the algorithms.